**PROBABILITY**

**SAMPLE SPACE:** The set of all possible outcomes of an experiment is known as sample space. It’s denoted by S.

**EVENT:** Any subset of the sample set is known as event.

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| **COUNTABLY INFINITE SET:**  **E.g.** | **UN-COUNTABLY INFINITE SET:**  **E.g.** | **UN-COUNTABLY FINITE SET:**  **E.g.** |

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| **PROBABILITY:** If the sample space S of an experiment consist of finitely many outcomes that are equally likely, then the probability of an event A is given by |  |

**GENERAL DEFINITION OF PROBABILITY:** Given a sample space S, with each event A of S, there is a number called the probability of A, such that the following axioms of probability are satisfied,

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|  |  | Exclusive Events: hence, |

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| **COMPLEMENT RULE:** For an eventand it’s Compliment in a sample space S,   |  |  | | --- | --- | |  |  | | **ADDITIONAL RULES FOR MUTUALLY EXCLUSIVE EVENTS:** n mutually exclusive events |

**ADDITIONAL RULE FOR ARBITRARY EVENTS:** For events and in the sample space S,

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**ADDITIONAL RULE FOR “n” ARBITRARY EVENTS:**

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| **CONDITIONAL PROBABILITY:** | **MULTIPLICATION THEOREM:** |
| **MUTUALLY EXCLUSIVE EVENTS** | **INDEPENDENT EVENTS** |
| Mutually exclusive events only have significance if we consider one particular performance of one particular experiment. | Independent Events can only be considered in multiple performances of the same experiment or different experiment or different experiments together. |
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| **Bay’s Rule:**  **Here,**  [Mutually Exclusive] |  |

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| **SAMPLING:** Randomly drawing objects from a given set of objects. | |
| **With Replacement** | **Without Replacement** |
| The object that was drawn at random is placed back to the given set and the set is mixed thoroughly. | The object that was drawn is put aside. |

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| **CARD CLASSIFICATIONS** | | | | **FACE CARDS** |
| 52 Cards | 26 Cards | 13 Heart Suits | A, 2, 3, …., 10, K, Q, J |
| 13 Diamond Suits | A, 2, 3, …., 10, K, Q, J | K, Q, J |
| 26 Cards | 13 Spades Suits | A, 2, 3, …., 10, K, Q, J |
| 13 Clubs Suits | A, 2, 3, …., 10, K, Q, J |

**RANDOM VARIABLE:** Random Variable is a function whose domain is a sample space and whose range is same set of real numbers.

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| **PROBABILITY MASS FUNCTION:**  And | |  |  | | --- | --- | | X | = Random Variable | | p(X) | = Probability of R. V. | |

Cumulative Distribution Function

**CONTINUOUS RANDOM VARIABLE (CRV):** A random variable x and it’s distribution are of continuous type if it’s cumulative distribution (CDF) F(x) is given by,

|  |  |
| --- | --- |
|  | Where Probability density function (PDF) of x  Relation Between CDF & PDF: |

**PROPERTIES OF CRV:**

1. **NON-NEGATIVITY:** The PDF is a Non-Negative function.
2. **NORMALIZATION:** The total area under the graph of the PDF is equal to Unity.

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| **EXPECTED VALUE OF RANDOM VARIABLE** | | |
| **DISCRETE RANDOM VARIABLE** | | **CONTINUOUS RANDOM VARIABLE** |
|  | |  |
| **PROPERTIES** | |  |
|  |  |
|  |  | If , |

**VARIANCE OR SECOND CENTRAL MOMENT OF A RANDOM VARIABLE:**

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| --- | --- |
| **PROPERTIES** |  |
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For independent Random variables,

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| **FUNCTION** |  | | |  |  |  |
| **Uniform Random Variable** | = 0 (Otherwise) | | |  |  |  |
| **Exponential Random Variable** | = 0 (Otherwise) | | |  |  |  |
| **Poisson Random Variable (Parameter )** | = 0 (Otherwise) | | |  |  |  |
| **Normal or Gaussian Distribution** |  | | |  | - |  |
| **Std. Normal or Gaussian Distribution** |  | | | 0 | - |  |
| For , Area =0.6827 | | For , Area = 0.9545 | For , Area = 0.996 | | | |

If is Normal Random variable with mean and variance then is also Normal random variable with and .

Question Can be twist in the form of integral problem.

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| **BERNOULLI RANDOM VARIABLE** | **BINOMIAL RANDOM VARIABLE** | |
| Suppose that a trail or an experiment whose outcome can be classified as either success or failure is performed. If we let X= 1, when the outcomes is success and X= 0, when outcome is failure, the probability mass function of X is given by,  , ,  Where | Suppose now that “n” independent trails each if which results in a success with probability and in a failure with probability are to be performed. If X represents the number of success that occurrence in the “n” trails then X is said to Binomial Random variable with parameters It’s Probability mass function given by, | |
| It’s Special case of binomial random variable with x = 0,1 |  |  |
|  |  | |

**NORMAL APPROXIMATION TO BINOMIAL RANDOM VARIABLE:**

When is large, a binomial random variable with parameters and will have approximately the same distribution as a normal random variable with the same mean and variance as the binomial. If X denotes the number of success that occur when independent trails each resulting in a success with probability are performed then for any ,

|  |  |
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|  | Here, Mean And Variance |

**POISSON APPROXIMATION TO BINOMIAL RANDOM VARIABLE:**

When is large and is small then binomial distribution is very closely approximated by Poisson distribution,

Poisson distribution is a limiting case of binomial distribution as and .

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|  | Here, is mean of Poisson Random variable.  Hence, |

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| **Exponential Random Variable (Parameter )** | **Poisson Random Variable** |
| In practice, It’s often arises as the distribution of the amount of time until some specific event occurs.   1. amount of time until an earthquake occurs. 2. amount of time until a new war breaks out. 3. amount of time until a telephone call you receive turns out to be a wrong number. | 1. The number of misprints on a page of book. 2. The number of people in a community who survive to age 100. 3. The number of Wrong telephone numbers that are dialled in a day. 4. The number of customers entering in a day in office. |

* If X and Y are independent Poisson R.V. with respective parameters and , then X+Y has a Poisson Distribution with parameter .
* If X and Y are independent Binomial R.V. with respective parameters and , then X+Y has a Poisson Distribution with parameter .
* If X and Y are independent Normal R.V. with respective parameters and then X+Y has a Poisson Distribution with parameter .

**SIGNIFICANCE VARIANCE OR SECOND CENTRAL MOMENT OF A RANDOM VARIABLE:**

It’s just description about how closely data is scattered about it’s mean value.

**STATISTICS**

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| **MEAN** | **MEDIAN** | **MODE** |
|  | It’s the middle value of the sample, if sample points are arranged in ascending order. If the number of elements in the sample is even, then the arrange of the two middle values can be taken as median. | The sample point that occurs with highest frequency. |
| Let X be a discrete random variable having the possible values ,  If | The median of probability distribution is the point at which the distribution function has the value of 0.5.  In case of a continuous distribution, the median corresponds to a point “x” which separates the density curve into two parts having equal areas. | For a discrete random variable, mode is the value x at which it’s probability mass function takes maximum value. For continuous random variable, mode is the x at which it’s probability density function has maximum value. So any peak is a mode. |

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|  | Mode = 3 Median – 2 Mean |